Higher Order Effects in Lepton-pair production

in

Relativistic Heavy-ion Collisions

Motivation to study lepton-pair production

- Relativistic colliders (SPS, RHIC, LHC)
- Strong electromagnetic fields
- Exotic particles (heavy leptons, vector mesons, Higgs etc.)
- Quark-gluon plasma
- Beam lifetimes (capture), detector background
- Perturbative and non-perturbative approach
- Impact parameter dependence
- Multiple-lepton pair production
- Implementation on parallel supercomputers

Experiments in Lepton-pair production

Free pair production: S + Au (200 A GeV)

Exp. 85 barn ∓ 25 % error. Scaleing $\propto Z^2$
Theory. 98 barn

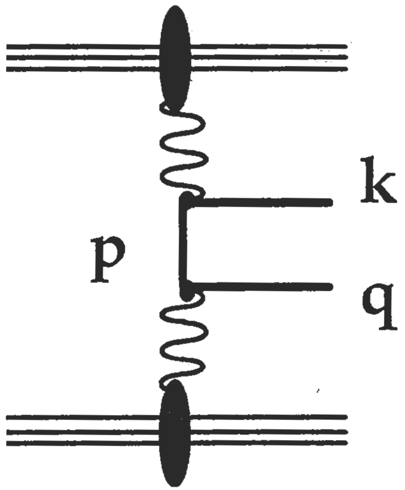
C. R. Vane, S. Datz, P. F. Dittner, H. F. Krause, C. Bottcher,
M. R. Strayer, R. Schuch, H. Gao, and R. Hutten
Phys. Rev. Lett. 69, 1911 (1992)

Pair production with capture: U + Au, Ag, Cu, Mylar (0.956 A GeV)

Exp. $Z^{2.8}$

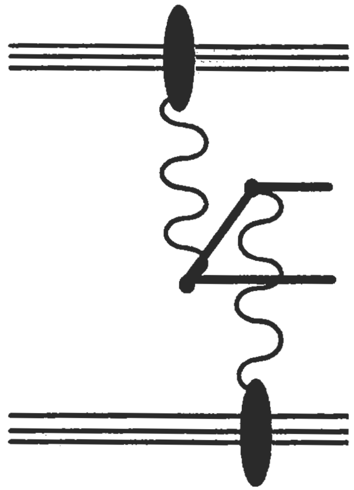
A. Belkacem, H. Gould, B. Feinberg, and W. E. Meyerhof
Phys. Rev. Lett. 71, 1524 (1993)

Two photon diagrams—by Feynman Monte Carlo



direct

a



exchange

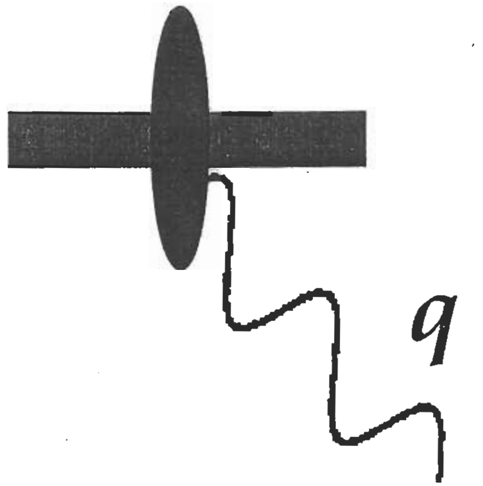
b

Classical Fields

$$\mathbf{A}^{\mu}(\vec{\mathbf{q}}) = \mathbf{A}_1^{\mu}(\vec{\mathbf{q}}) + \mathbf{A}_2^{\mu}(\vec{\mathbf{q}})$$

$$\mathbf{A}_{1,2}^{\mu}(\vec{\mathbf{q}}) = -8\pi^2 \mathbf{Z}_{1,2}\gamma^2 \mathbf{u}_{1,2}^{\mu} \frac{\delta(\mathbf{q}_0 \pm \beta\,\mathbf{q}_z)}{\mathbf{q}_z^2 + \gamma_\perp^2} \mathbf{e}^{\mp i\vec{\mathbf{q}}_\perp \cdot \vec{\mathbf{b}}/2}$$

$\mathbf{A}_1^{\mu}(\vec{\mathbf{q}})$

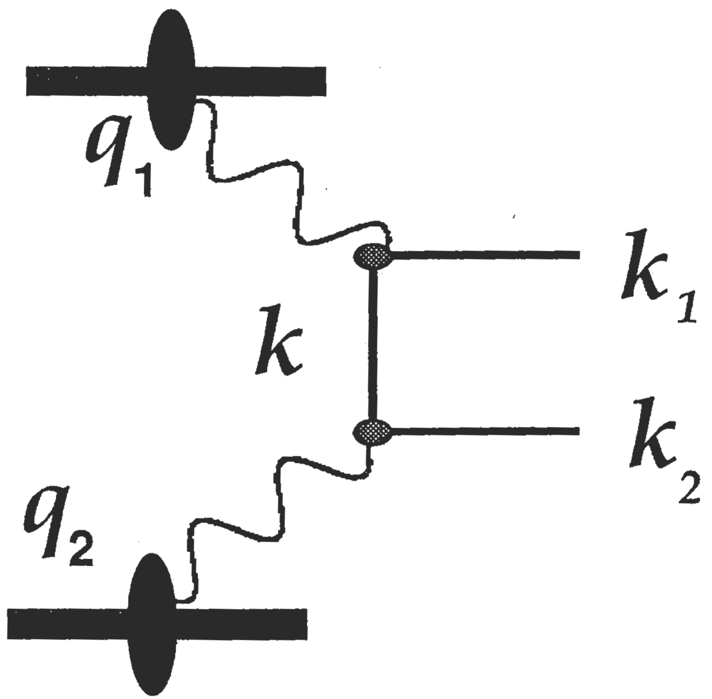


relates frequencies in the field to the "boost"

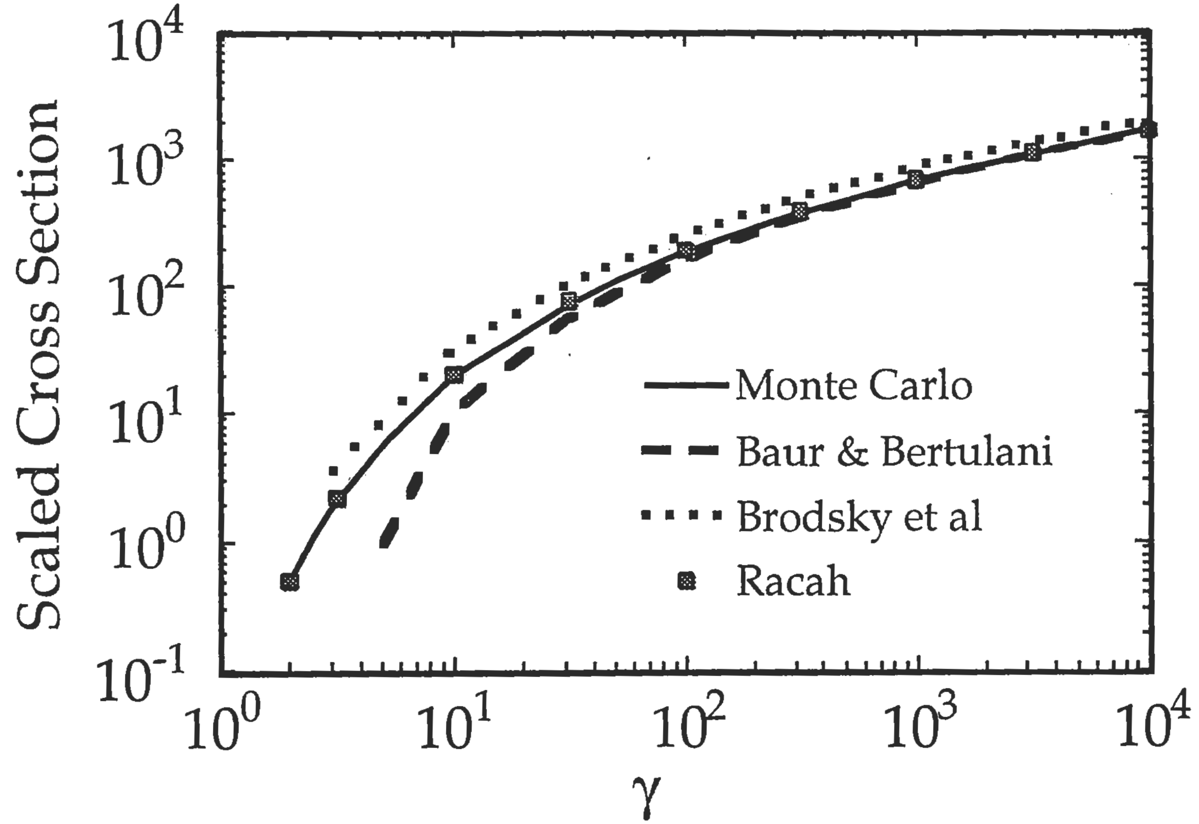
Cross Section

$$\sigma = \frac{1}{4\beta^2} \sum_{\sigma_k \sigma_q} \int \frac{d^3k d^3q d^2p_\perp}{(2\pi)^8} \left| A^{(+)}(k,q;\vec{p}_\perp) + A^{(-)}(k,q;\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp) \right|^2$$

Eight dimensional regular integral



Electron Pair Cross Section Vs Beam Energy



Impact parameter dependence

In the interaction picture the time-evolved vacuum state

$$S|0\rangle = \lim_{t\to\infty} K_0(0,t)K(t,-t')|0\rangle$$

Inclusive pair cross section

$$\sigma = \int d^2b \sum_{k>0}\sum_{q<0} \left|\langle \chi_k^{(+)} | S | \chi_q^{(-)} \rangle\right|^2$$

The transition matrix element for direct Feynman diagram

$$\langle \chi_k^{(+)} | S | \chi_q^{(-)} \rangle = \frac{i}{2\beta}\int \frac{d^2p_\perp}{(2\pi)^2} \exp\left\{ i\left[\vec{p}_\perp - \left(\frac{\vec{k}_\perp + \vec{q}_\perp}{2}\right)\right]\cdot \vec{b} \right\} \mathcal{A}^{(+)}(k,q:\vec{p}_\perp).$$

Including both the direct and crossed Feynman diagrams,

$$\frac{d\sigma}{db} = \frac{1}{4\beta^2} \sum_{\sigma_k,\sigma_q} \int \frac{d^3k d^3q d^2p_\perp d^2p'_\perp}{(2\pi)^9} b J_0(b|\vec{p}_\perp - \vec{p}'_\perp|)$$
$$\times [\mathcal{A}^{(+)}(k,q:\vec{p}_\perp) + \mathcal{A}^{(-)}(k,q:\vec{k}_\perp + \vec{q}_\perp - \vec{p}_\perp)]$$
$$\times [\mathcal{A}^{(+)}(k,q:\vec{p}'_\perp) + \mathcal{A}^{(-)}(k,q:\vec{k}_\perp + \vec{q}_\perp - \vec{p}'_\perp)]^*$$

where

$$\mathcal{A}^{(+)}(k,q:\vec{p}_\perp) = F(\vec{k}_\perp - \vec{p}_\perp:\omega_1)F(\vec{p}_\perp - \vec{q}_\perp:\omega_2)\mathcal{T}_{kq}(\vec{p}_\perp:+\beta)$$
$$\mathcal{A}^{(-)}(k,q:\vec{p}_\perp) = F(\vec{k}_\perp - \vec{p}_\perp:\omega_2)F(\vec{p}_\perp - \vec{q}_\perp:\omega_1)\mathcal{T}_{kq}(\vec{p}_\perp:-\beta)$$

and $F(\vec{q},\omega)$ **is the scalar part of the EM field of the moving ions in momentum space:**

$$F(\vec{q}:\omega) = \frac{4\pi Z\gamma^2\beta^2}{\omega^2 + \beta^2\gamma^2 q^2} G_E(q^2) f_Z(q^2).$$

The frequencies ω_1 and ω_2 of the virtual photons are fixed by energy conservation at the vertex where the photon is absorbed.

$$\omega_1 = \frac{E_q^{(-)} - E_k^{(+)} + \beta(q_z - k_z)}{2},$$

$$\omega_2 = \frac{E_q^{(-)} - E_k^{(+)} - \beta(q_z - k_z)}{2}.$$

The quantity $\mathcal{T}$ contains the propagator of the intermediate lepton and the matrix elements for the coupling of the photon to the leptons

$$\mathcal{T}_{kq}(\vec{p}_\perp : \beta) = \sum_{s} \sum_{\sigma_p} [E_p^{(s)} - (\frac{E_k^{(+)} + E_q^{(-)}}{2}) + \beta(\frac{k_z - q_z}{2})]^{-1}$$

$$\times \left\langle u_{\sigma_k}^{(+)} \right| (1 - \beta\alpha_z) \left| u_{\sigma_p}^{(s)} \right\rangle \left\langle u_{\sigma_p}^{(s)} \right| (1 + \beta\alpha_z) \left| u_{\sigma_q}^{(-)} \right\rangle.$$

Numerical Techniques

OBSTACLE:

$J_0(b|\vec{p}_\perp - \vec{p}_\perp|)$ **is a rapidly oscillating function.**

SOLUTION:

We divide the integration according to

$$\frac{d\sigma}{db} = \int_0^\infty dq\, q\, b\, J_0(qb)\mathcal{F}(q)$$

$\mathcal{F}(q)$ **is given by a nine-dimensional integral:**

$$\mathcal{F}(q) = \frac{\pi}{8\beta^2} \sum_{\sigma_k} \sum_{\sigma_q} \int_0^{2\pi} d\varphi_q \int \frac{dk_z dq_z d^2k_\perp d^2K d^2Q}{(2\pi)^{10}}$$

$$\times F[\frac{1}{2}(\vec{Q}-\vec{q});\omega_1] F[-\vec{K};\omega_2] F[\frac{1}{2}(\vec{Q}+\vec{q});\omega_1] F[-\vec{q}-\vec{K};\omega_2]$$

$$\times \left\{ \mathcal{T}_{kq}[\vec{k}_\perp - \frac{1}{2}(\vec{Q}-\vec{q});+\beta] + \mathcal{T}_{kq}[\vec{k}_\perp - \vec{K};-\beta] \right\}$$

$$\times \left\{ \mathcal{T}_{kq}[\vec{k}_\perp - \frac{1}{2}(\vec{Q}+\vec{q});+\beta] + \mathcal{T}_{kq}[\vec{k}_\perp + \vec{q}_\perp - \vec{K};-\beta] \right\}$$

where we changed variables to :

$$\vec{p}_\perp = \vec{k}_\perp - \frac{1}{2}(\vec{Q}-\vec{q}),$$

$$\vec{q}_\perp = \vec{k}_\perp - \frac{1}{2}(\vec{Q}-\vec{q}) + \vec{K},$$

$$\vec{p}'_\perp = \vec{k}_\perp - \frac{1}{2}(\vec{Q}+\vec{q}).$$

Monte Carlo Integration

$$\mathcal{F}(q) = F_0 \int f(x_1, x_2, \ldots, x_8, q)\, dx_1 dx_2 \ldots dx_8$$

$$\vec{x} = \{\xi, \eta, k, \varphi_q, \theta_Q, \varphi_Q, \theta_K, \varphi_K\}$$

$$\begin{pmatrix} k_z \\ q_z \end{pmatrix} = \gamma e^{\xi} \begin{pmatrix} \cos\eta \\ \sin\eta \end{pmatrix}$$

$$\begin{pmatrix} Q_x \\ Q_y \end{pmatrix} = a_Q \tan\theta_Q \begin{pmatrix} \cos\varphi_Q \\ \sin\varphi_Q \end{pmatrix}$$

$$\begin{pmatrix} K_x \\ K_y \end{pmatrix} = a_K \tan\theta_K \begin{pmatrix} \cos\varphi_K \\ \sin\varphi_K \end{pmatrix}$$

$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = q \begin{pmatrix} \cos\varphi_q \\ \sin\varphi_q \end{pmatrix}$$

$$\begin{pmatrix} k_x \\ k_y \end{pmatrix} = \begin{pmatrix} k \\ 0 \end{pmatrix}$$

Monte Carlo methods reduce the equation to a summation:

$$\mathcal{F}(q) = \sum f(x_{i1}, x_{i2}, \ldots, x_{i8}, q)\Delta^8 x$$

$\Delta^n x$ **is the volume element.**

For a finite volume V,

$$\mathcal{F}(q) = \frac{V}{N}\sum f(x_{i1}, x_{i2}, \ldots, x_{i8}, q)\Delta^8 x$$

We can monitor the fluctuation of the result and, according to the central limit theorem for large values of N, we can write the associated error:

$$\delta^2 = (\langle \mathbf{f}^2 \rangle - \langle \mathbf{f} \rangle^2) / \mathbf{N}$$

where

$$\langle \mathbf{f}^2 \rangle = \frac{1}{\mathbf{N}}\sum \mathbf{f}^2(x_{i1}, x_{i2}, \ldots, x_{i8}, q)$$

$$\langle \mathbf{f} \rangle = \frac{1}{\mathbf{N}}\sum \mathbf{f}\,(x_{i1}, x_{i2}, \ldots, x_{i8}, q).$$

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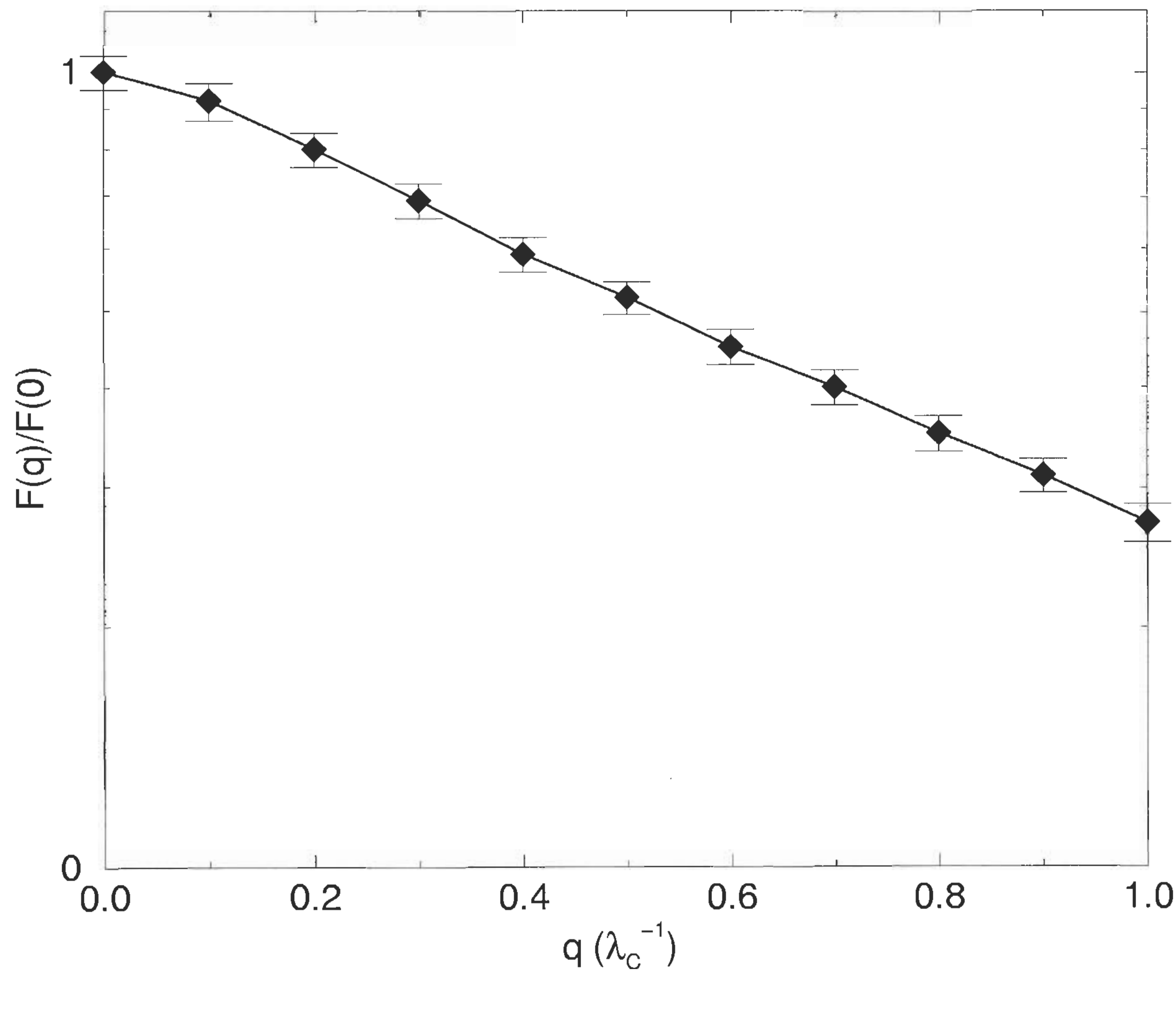


$$F(0) = \sigma_T$$

$$\sigma_T = C_\infty \sigma_0 \ln^3(\gamma)$$

$$F(q) = F(0)\, e^{-a\,q}$$

$$= \sigma_T e^{-a\,q}$$

$$a \cong 1.35\ \lambda_C$$

M. C. Guclu, Nucl. Phys. A, Vol. 668, 207-217 (2000)

$$\frac{d\sigma}{db} = \sigma_T \int_0^\infty dq\, q\, b\, J_0(q\, b)\, e^{-q\, b}$$

$$= \sigma_T \int_0^\infty dq\, q\, b \frac{2}{\pi} \int_0^1 \frac{\cos(qbt)}{\sqrt{1-t^2}} e^{-a\, q}\, dt$$

$$= \sigma_T \frac{a\, b}{(a^2 + b^2)^{\frac{3}{2}}}$$

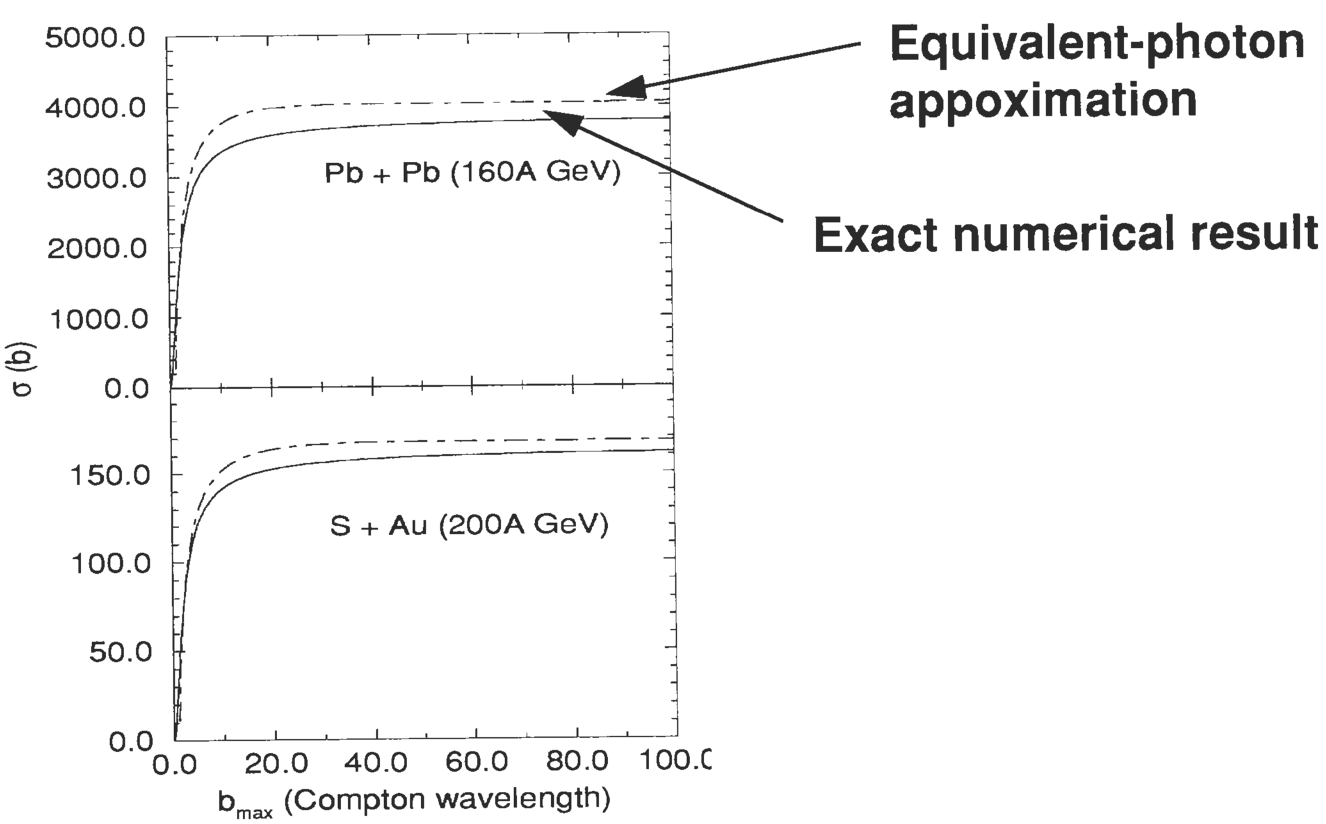
Two-photon method:

$$P(b)=\frac{1}{2\pi b}\frac{d\sigma}{db}=\frac{1}{2\pi}C_{\infty}\,\lambda_C^2\,Z_1^2\,Z_2^2\,\alpha^4\ln^3(\gamma)\frac{a}{(a^2+b^2)^{\frac{3}{2}}}$$

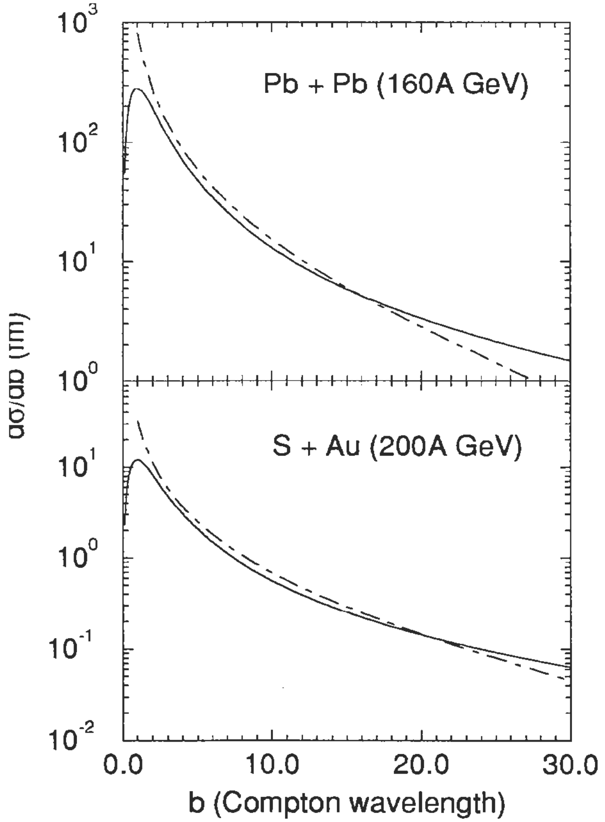
Equivalent-photon approximation :

$$P(b)\approx\frac{14}{9\pi^2}Z_1^2\,Z_2^2\,\alpha^4\left[\frac{\lambda_C}{b}\right]^2\ln^2\left(\frac{\gamma_{lab}\delta\,\lambda_C}{2b}\right)+\Delta(Z)$$

Integrated cross section



Differential cross section



$$\frac{d\sigma}{db} = \int_0^\infty dq\, q\, b\, J_0(qb)\, \mathcal{F}(q)$$

Multiple-pair production

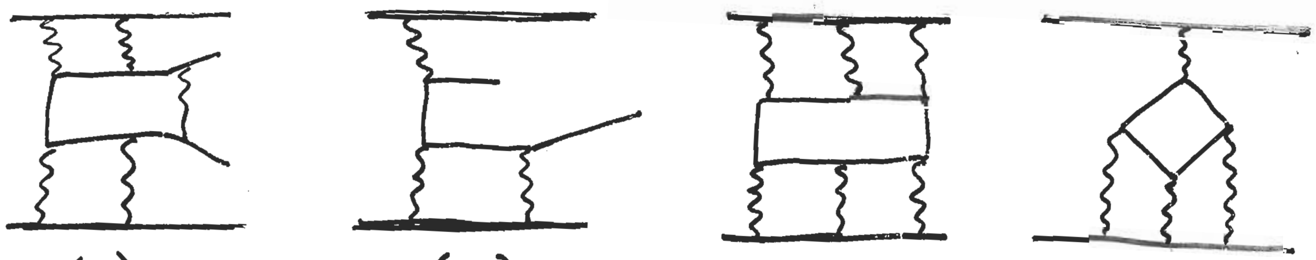
WHY ?

In most lowest-order perturbative calculations, pair production probability violates unitarity for small impact parameters and extreme energies.

SOLUTION ?

Summation of the classes of diagrams resulting from independent pair approximation can be used to restore unitarity to the lowest-order perturbation theory.

Diagrams which are not included.

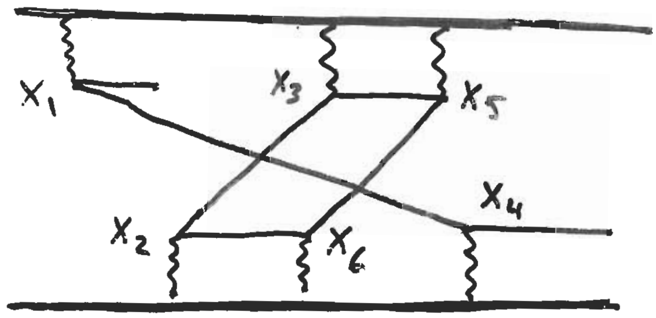


1) The model interaction contains only the electromagnetic field of the ions. There is no electromagnetic interaction between the created lepton and anti-lepton. Thus • the diagram of (a), or any diagram which would contain it as a part of a larger diagram, is not included.

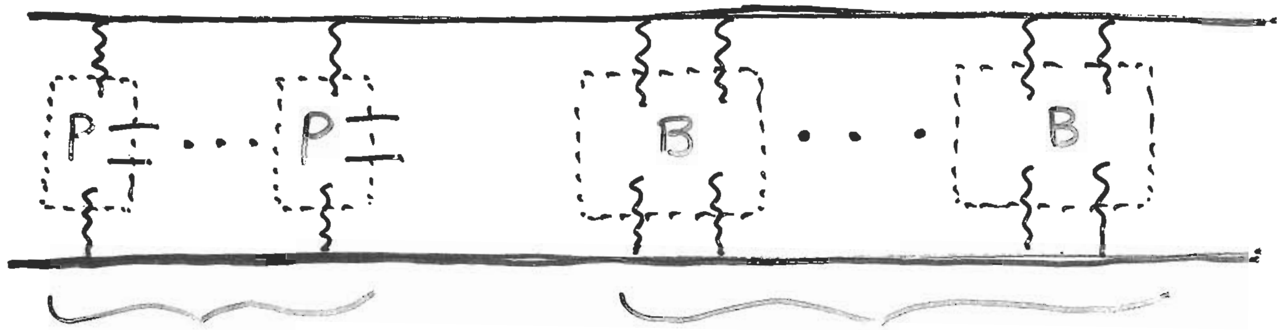
2) The final state interaction of a created lepton or anti-lepton with the electromagnetic field of the ions is neglected. Thus the diagram of (b) or any diagram which would contain it as a part of a larger diagram, is not included.

3) Fermion loops are included. However, only those which have exactly two vertices attached to each of the ions are included. Thus the diagrams of (c) and (d) or any diagram which would conta them as a part of a larger diagram, are not included.

An example of a diagram which is included



A diagram can be rearranged into pairs and boxes.



$$\frac{\hat{\alpha}_2(\vec{p},\vec{p}^+)}{2!} = \langle \vec{p}\,\sigma_p | S_b | \vec{p}^+ \sigma_{p^+} \rangle\, a_p^+ a_{p^+}^+$$

The S matrix operator for creating a lepton-antilepton pair with moment $\vec{p}$ and $\vec{p}^+$, respectively.

$\frac{\alpha_4}{4!} \longrightarrow$ S-matrix for the box diagram.

The S matrix having n boxes and N pairs of lepto[ns]
with momenta $|\vec{p}_1 \vec{p}_1^{\,+}\rangle, |\vec{p}_2 \vec{p}_2^{\,+}\rangle, \ldots, |\vec{p}_N \vec{p}_N^{\,+}\rangle$ is

$$S_{N,n}(\vec{p}_1 \vec{p}_1, \ldots, \vec{p}_N \vec{p}_N^{\,+}) = \frac{1}{(2N+4n)!} \binom{2N+4n}{2n} \frac{(2N)!}{(2!)^N N!} \frac{(4n)!}{(4!)^n n!}$$

$$\times \alpha_4^n \hat{\alpha}_2(\vec{p}_1 \vec{p}_1^{\,+}) \hat{\alpha}_2(\vec{p}_2 \vec{p}_2^{\,+}) \cdots \hat{\alpha}_2(\vec{p}_N \vec{p}_N^{\,+})$$

The S-matrix for the production of N pairs,

$$S_N = \sum_{n=0}^{\infty} S_{N,n} = e^{\left(\frac{\alpha_4}{4!}\right)} \frac{1}{N!} \left(\frac{\hat{\alpha}_2(\vec{p}_1 \vec{p}_1^{\,+})}{2!}\right) \cdots \left(\frac{\hat{\alpha}_2(\vec{p}_N \vec{p}_N^{\,+})}{2!}\right)$$

The probability for N pairs

$$P_N(b) = \frac{e^{\left[2\mathrm{Re}\left(\frac{\alpha_4}{4!}\right)\right]}}{N!} \left\langle 0 \left| \frac{\hat{\alpha}_2^{+}}{2!} \frac{\hat{\alpha}_2}{2!} \right| 0 \right\rangle^N$$

$$-2\mathrm{Re}\left(\frac{\alpha_4}{4!}\right) = \left\langle 0 \left| \frac{\hat{\alpha}_2^{+}}{2!} \frac{\hat{\alpha}_2}{2!} \right| 0 \right\rangle$$

$$= \mathcal{P}(b) \equiv$$ Perturbative probability of creating a single pa[ir]

To arrive at the total probability of creating N pair[s]
we must integrate over the impact parameter,

$$\sigma_{N\,\text{pair}} = \int d^2b \, P_N(b), \quad N = 1, 2, 3, \ldots$$

$$\sigma_{\text{pair}} = \sum_{N=1} \sigma_{N\,\text{pair}}$$ Total cross section

Poisson Distribution

$$P_N(b) = \frac{\mathcal{P}(b)^N \exp[-\mathcal{P}(b)]}{N!}$$

where N denotes N-pair production and

$$\mathcal{P}(b) = \sum_{k>0}\sum_{q<0} \left| \left\langle \chi_k^{(+)} \middle| S \middle| \chi_q^{(-)} \right\rangle \right|^2$$

is the lowest-order perturbation result for the pair-production probability and can be obtained :

$$\sigma = \int d^2b\,\mathcal{P}(b) \;\Rightarrow\; \mathcal{P}(b) = \frac{1}{2\pi b}\frac{d\sigma}{db}$$

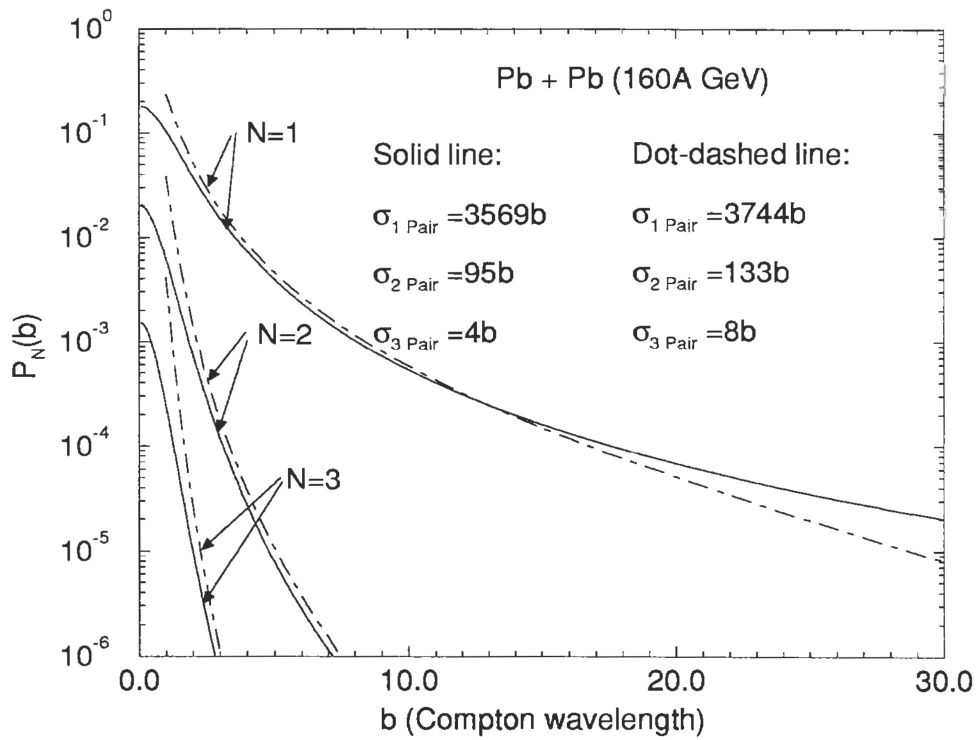
N-pair cross section can be obtained :

$$\sigma_{N\,pair} = \int d^2b\,P_N(b), \quad N = 1,2,\ldots$$

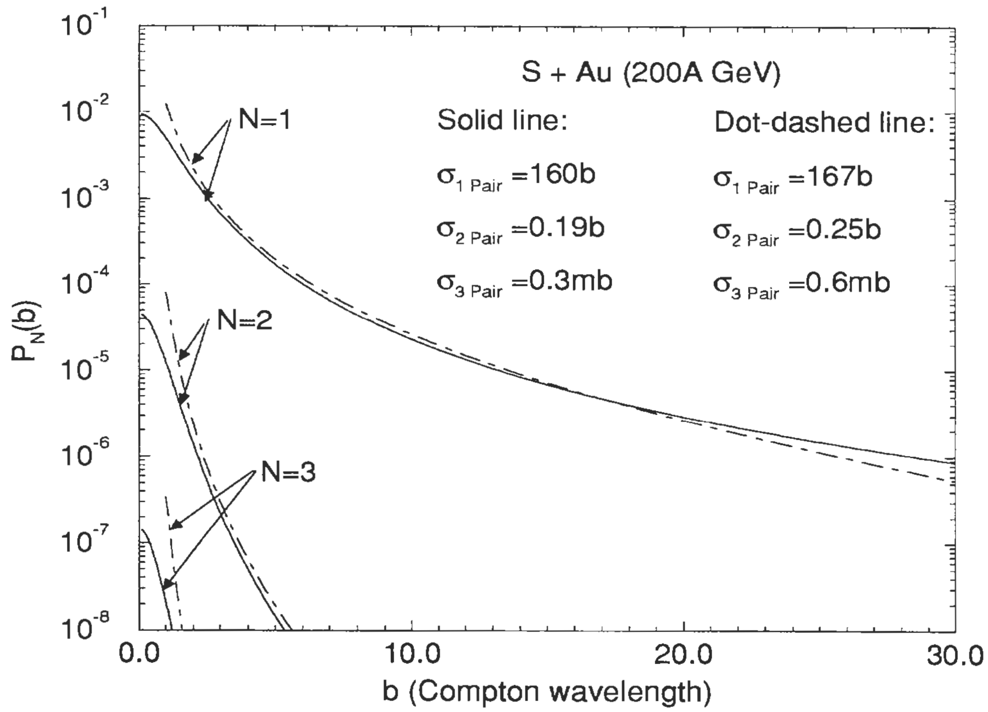
The total cross section for producing any number of pairs:

$$\sigma_{pair} = \sum_{N=1} \sigma_{N\,pair}\,.$$

N-pair Probability Distribution for Pb + Pb



N-pair Probability Distribution for S + Au



CONCLUSIONS :

(i) We have found a technique that is capable of calculating the semiclassical two-photon mechanism for lepton-pair production as a function of the impact parameter.

(ii) By using the two-photon diagrams as the central ingredient, we calculate the multiple-pair production cross section.

(iii) Single-pair production is the dominant part of the total production cross section as expected.

(iv) At these energies, nonperturbative effects remain relatively small.

(v) Exact Monte Carlo methods does a fairly good job of predicting the total pair-production cross-section for all impact parameters.

(vi) Prediction of exotic particles.