The Quark-Mass Dependence of T_C in QCD: Working up from $m_q = 0$ or down from $m_q = \infty$?

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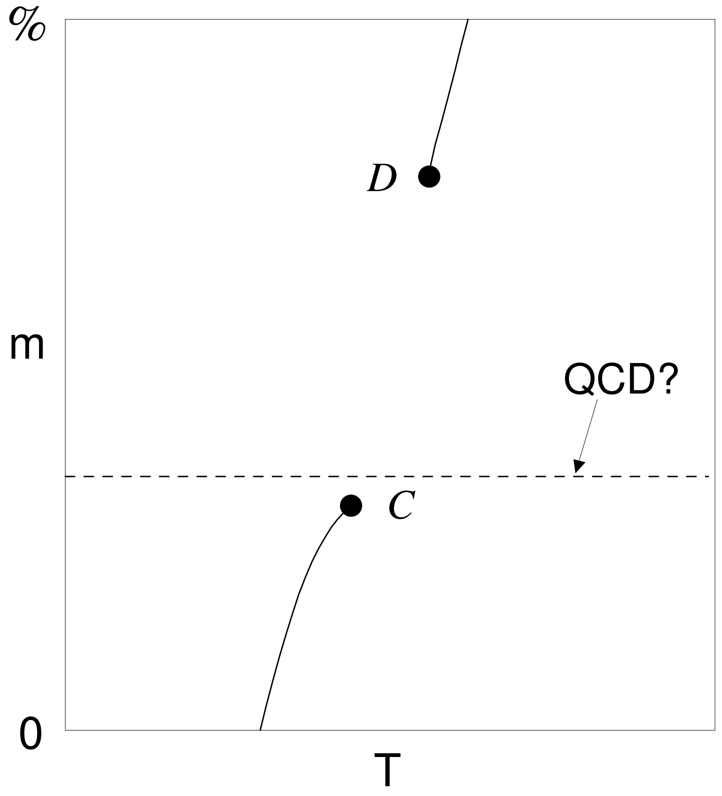
Motivation



• Schematic phase diagram in the temperature vs. quark mass plane. C is the chiral critical point, D the deconfining critical point.

• Part 1: compute T_c in the $O(4)$-model and compare with lattice data.

• Part 2: compute T_c and find D in the Polyakov loop model and compare the results with lattice data.

Gavin, Gocksch and Pisarski, Phys. Rev. D **49**, (1994).

Motivation: the $O(4)$ linear sigma model

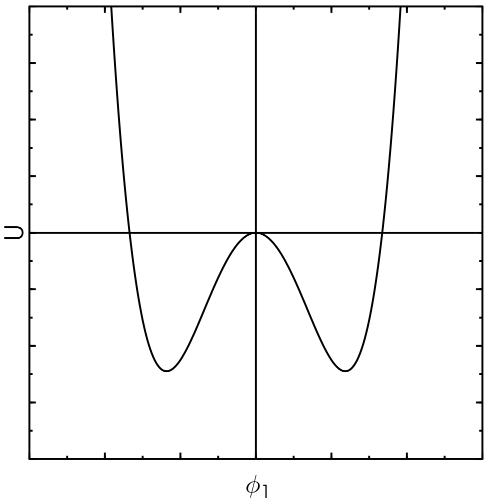
• The Lagrangian of the $O(4)$ model

$$\mathcal{L}(\bar{\phi}) = \frac{1}{2}\left(\partial_\mu \underline{\phi} \cdot \partial^\mu \underline{\phi} - m^2\, \underline{\phi} \cdot \underline{\phi}\right) - \frac{\lambda}{4}\left(\underline{\phi} \cdot \underline{\phi}\right)^2 + H \phi_1 \quad .$$

• For $H = 0$ and $m^2 < 0$ the symmetry is spontaneously broken to $O(3)$, with 3 Goldstone bosons (π^+,π^-, and π^0)

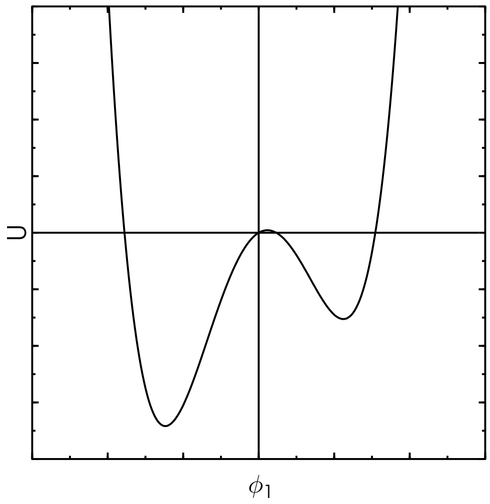
• For $H \neq 0$ and $m^2 < 0$ the symmetry is also explicitly broken, to give a mass to the 3 Goldstone bosons

mexican hat potential ($m^2 < 0\,,\; H = 0$)



3–Goldstone bosons

“tilted” mexican hat potential ($m^2 < 0\,,\; H \neq 0$)



3–Pseudogoldstone bosons

The effective potential of the $O(4)$-model

- $O(4) \simeq SU(2)_r \times SU(2)_\ell$
- We use the imaginary time formalism to introduce the temperature dependency.

$\int_k f(k) \equiv T \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^2} f(2\pi i m T, k)$

- $\phi \equiv (\sigma, \vec{\pi})$

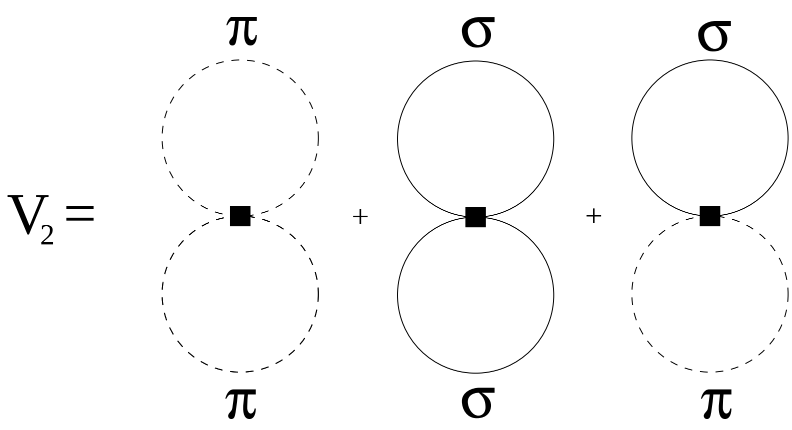
$$
\begin{aligned}
V(\phi, \mathcal{S}, \mathcal{P}) &= \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} \phi^4 - H\phi \\
&+ \frac{1}{2} \int_k \left[\ln \mathcal{S}^{-1}(k) + S^{-1}(k;\phi)\, \mathcal{S}(k) - 1\right] \\
&+ \frac{3}{2} \int_k \left[\ln \mathcal{P}^{-1}(k) + P^{-1}(k;\phi)\, \mathcal{P}(k) - 1\right] \\
&+ V_2[\phi, \mathcal{S}, \mathcal{P}]
\end{aligned}
$$

- V_2 is the sum of the two-particle irreducible diagramms

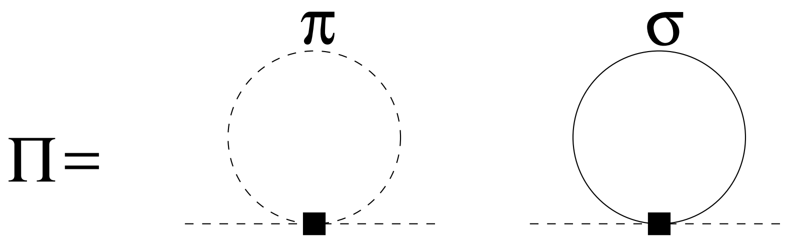
Luttinger and Ward, Phys. Rev. **118**, (1960),

Cornwall, Jackiw and Tomboulis, Phys. Rev. D **10**, (1974).

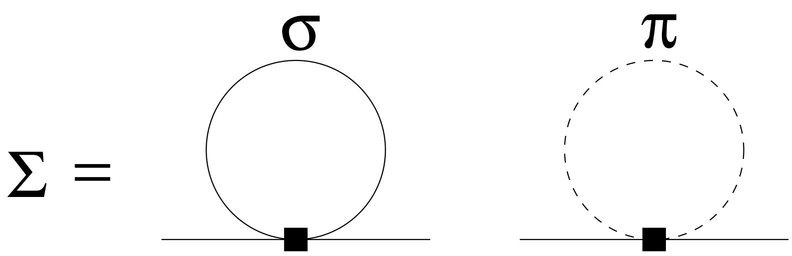
The Hartree approximation



- $$\frac{\delta V[\phi,\mathcal{S},\mathcal{P}]}{\delta \mathcal{P}} = 0 \Rightarrow \mathcal{P}^{-1}(k) \quad = \quad P^{-1}(k,\phi) + \Pi[\mathcal{S},\mathcal{P}]$$



- $$\frac{\delta V[\phi,\mathcal{S},\mathcal{P}]}{\delta \mathcal{S}} = 0 \Rightarrow \mathcal{S}^{-1}(k) \quad = \quad S^{-1}(k,\phi) + \Sigma[\mathcal{S},\mathcal{P}]$$



- $$\frac{\delta V[\phi,\mathcal{S},\mathcal{P}]}{\delta \phi} = 0 \Rightarrow H = m^2\phi + \lambda\phi^3 + 3\lambda \int_q [\mathcal{S}(q) + \mathcal{P}(q)]$$

The parameters of the $O(4)$-model

- The parameters:

$$H = m_\pi{}^2 f_\pi \;, \quad \lambda = \frac{m_\sigma{}^2 - m_\pi{}^2}{2 f_\pi{}^2} \;, \; m^2 = -\frac{m_\sigma{}^2 - 3 m_\pi{}^2}{2} - 6\lambda Q_\mu(m_\pi)$$

with $Q_\mu(M) \equiv \frac{1}{(4\pi)^2}\left[M^2 \ln\frac{M^2}{\mu^2} - M^2 + \mu^2\right]$

- m_π: tree-level mass of the pions
- f_π: tree-level decay constant of the pions
- m_σ: tree-level mass of the sigma

- The dependenc on the quark mass m_q:

$$\begin{aligned} m_\pi{}^2(m_q) &= c_1\, m_q \\ f_\pi(m_\pi) &= c_2 + c_3\, m_\pi{}^2 \\ m_\sigma(m_\pi) &= c_4 + c_5\, m_\pi{}^2 \end{aligned}$$

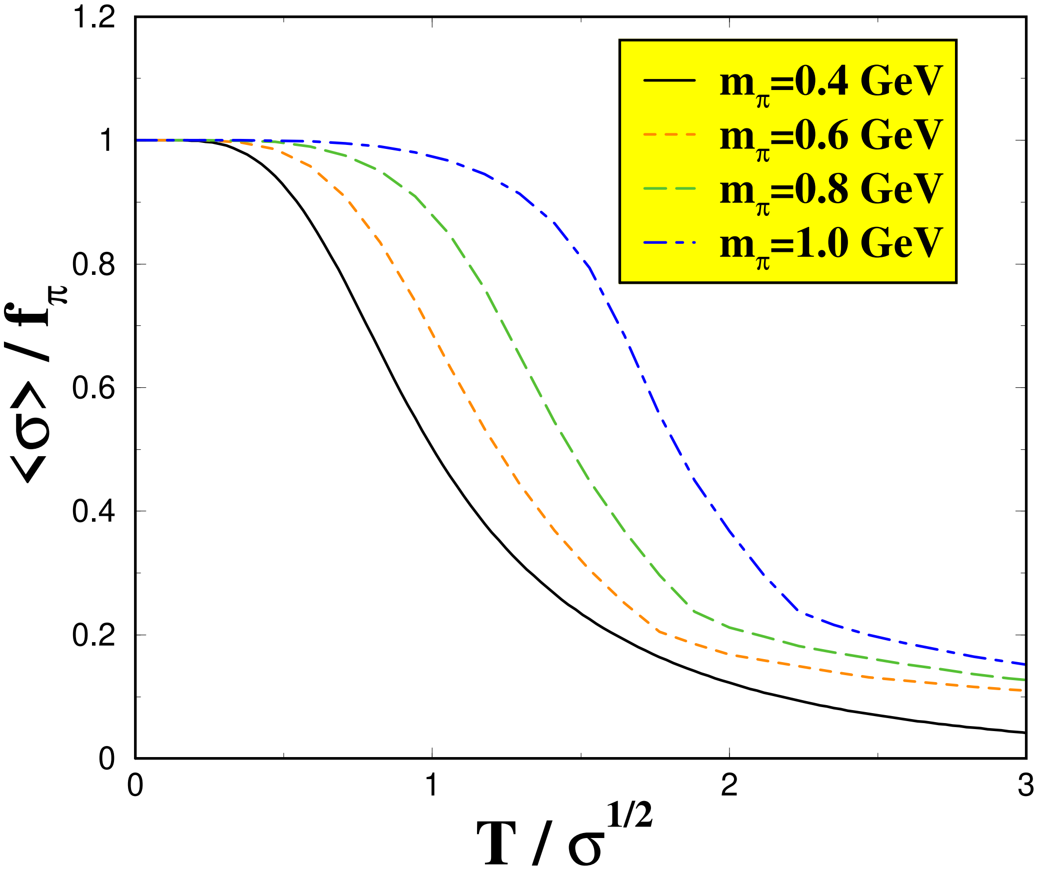
- $m_\pi(m_q)$, $f_\pi(m_\pi)$; $0.4\,\mathrm{GeV} \leq m_\pi \leq 1\,\mathrm{GeV}$:

Chiu and Hsieh, Nucl. Phys. B **673**, (2003).

- $m_\sigma(m_\pi)$:

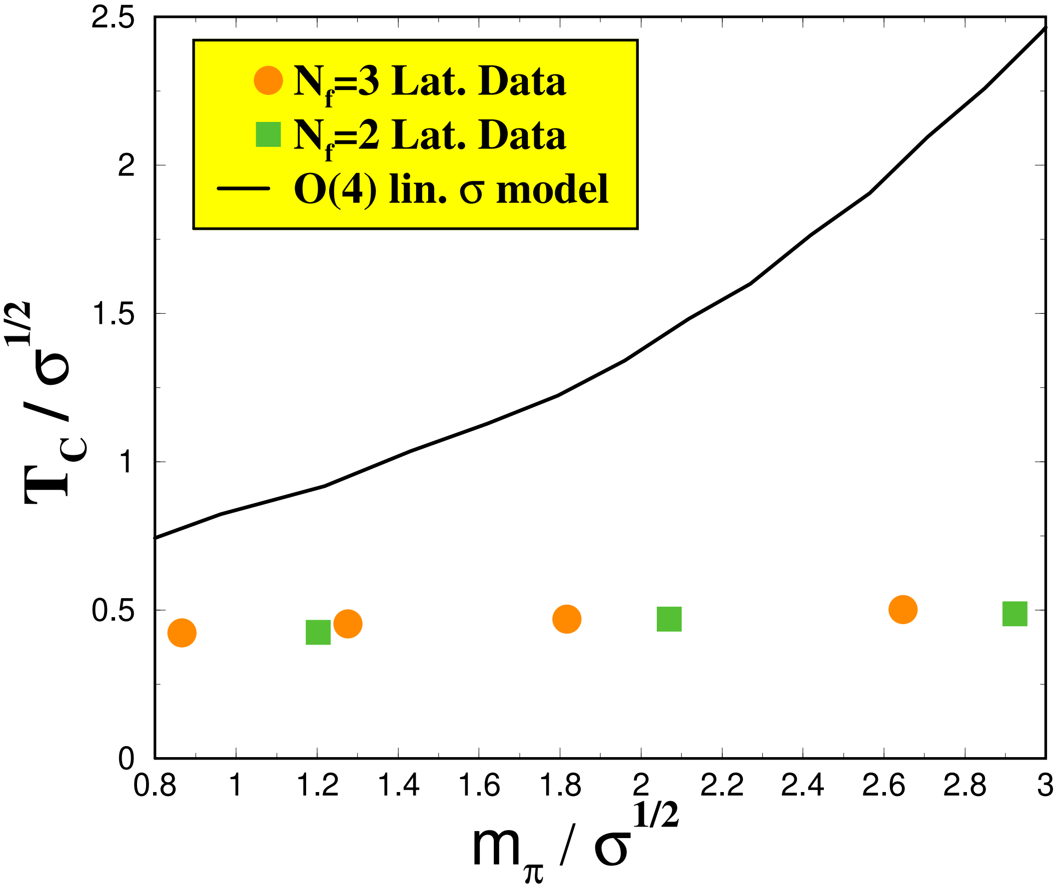
Kunihiro, Muroya, Nakamura, Nonaka, Sekiguchi and Wada [SCALAR Collaboration], arXiv:hep-ph/0310312.

The scalar condensate



• The increasing of the the pion mass leads to a broadening of the “phase transition”.

The cross over temperature T_c as a function of m_π



- $\mathrm{H}(m_\pi = 1\,\mathrm{GeV}) \approx 10 * \mathrm{H}(m_\pi = 0.4\,\mathrm{GeV})$
- The scale is given by the zero-temperature string tension $\sqrt{\sigma} \simeq 425$ MeV.

Karsch, Laermann and Peikert, Nucl. Phys. B **605**, (2001).

The Polyakov loop

• In the Limit $m_q \to \infty$ the quarks decouple and drop out of the theory (pure gauge theory).

• The order parameter for such theory is the expectation value of the Polyakov loop

$$\ell = \frac{1}{N_c} \operatorname{tr} \mathcal{P} \exp \left(ig \int_0^{1/T} A_0(\vec{x}, \tau)\, d\tau \right) \; .$$

• N_c is the number of colors.

• g is the gauge coupling.

• A_0 is the temporal component of the gauge field.

$$\langle \ell \rangle = 0 \; , \; T < T_c \quad ; \quad \langle \ell \rangle > 0 \; , \; T > T_c$$

Pisarski, Phys. Rev., D**62**, (2000);
Scavenius, Dumitru, and Jackson, Phys. Rev. Lett. **87**, (2001).

The effective potential for the Polyakov loop model

- The effective potential for 3 colors and finite m_π

$$V(\ell) = -b_1 \frac{\ell + \ell^*}{2} - \frac{b_2}{2}|\ell|^2 - \frac{b_3}{3}\frac{\ell^3 + \ell^{*3}}{2} + \frac{1}{4}\left(|\ell|^2\right)^2$$

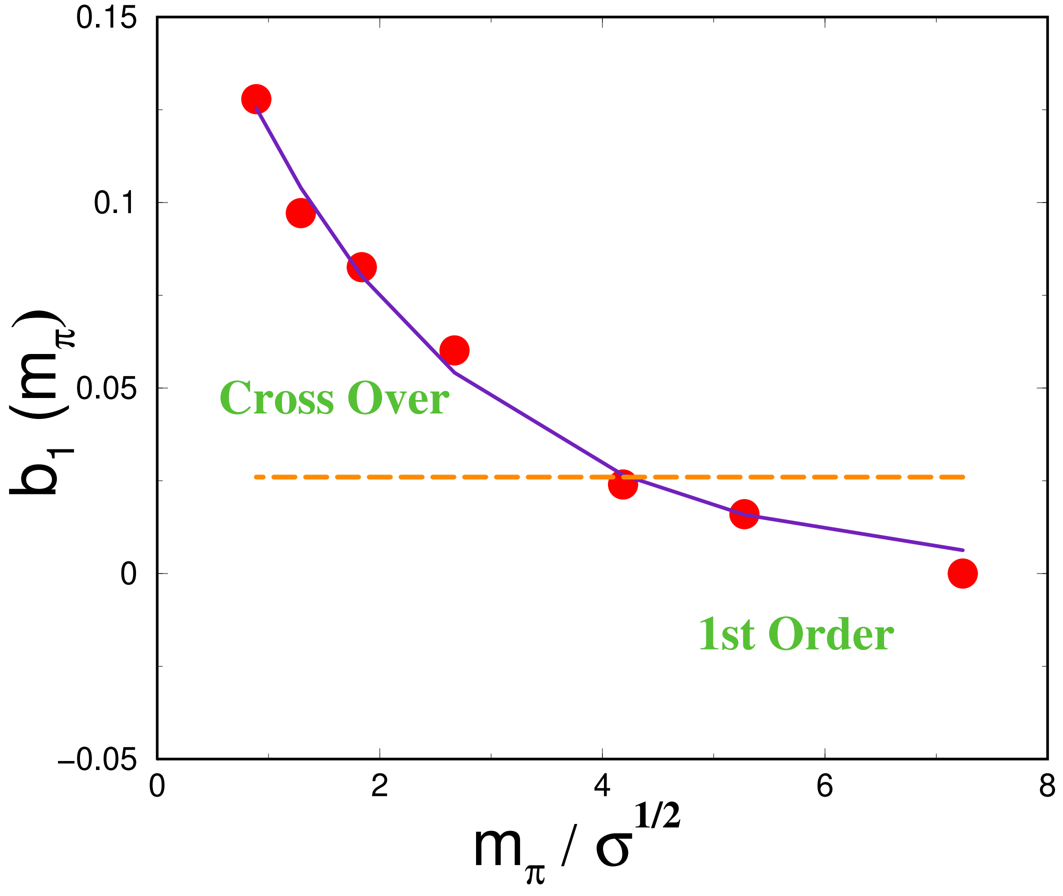
- $b_1 \sim \exp(-m_\pi)$ breaks the $Z(3)$-symmetry explicitly, in this work b_1 is obtained by fitting the T_c to the lattice data.

- $b_2(T)$ is the mass term and the only temperature dependend parameter in this model.

- $b_3 \approx 0.9$ is obtained by fitting the effective potential to the pressure and energy density of the pure gauge theory with three colors.

Dumitru and Pisarski, Phys. Lett. **B504**, (2001);
Scavenius, Dumitru and Lenaghan, Phys. Rev. C **66**, (2002)

The parameter b_1 as a function of m_π



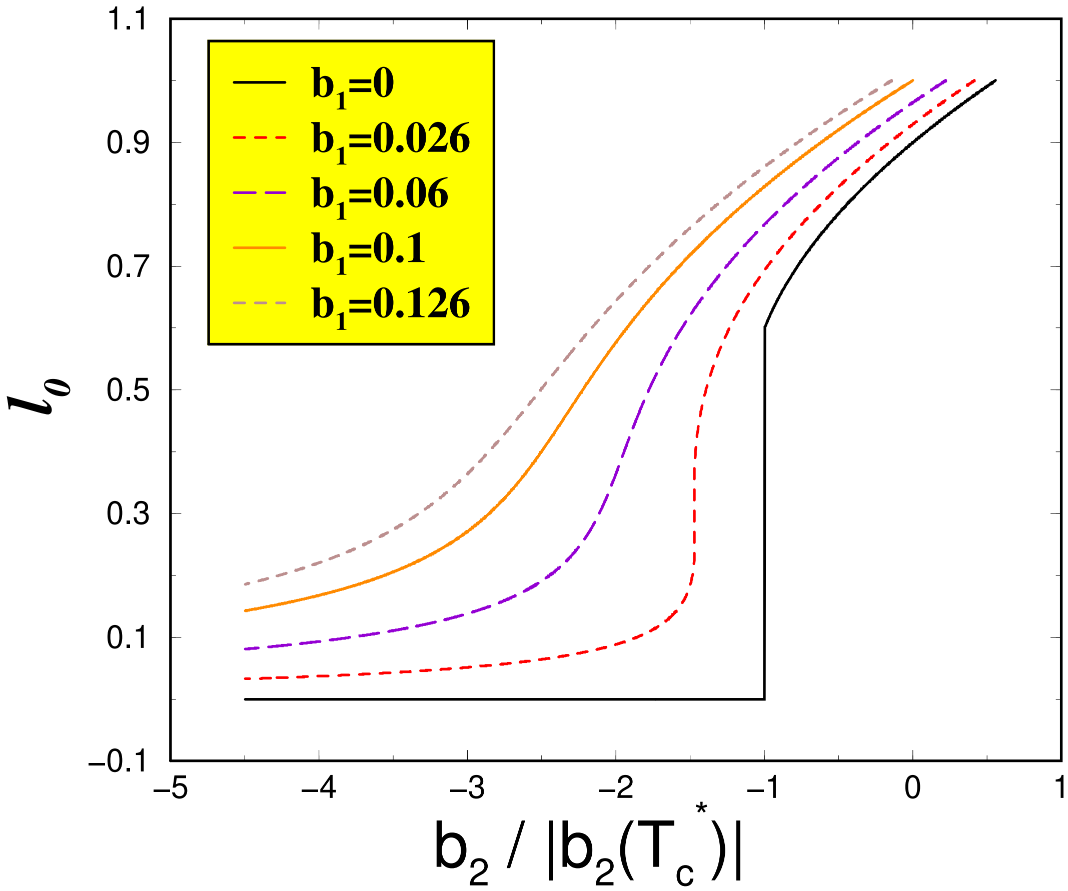
- $b_1(m_\pi = 1\,\mathrm{GeV}) \approx 2 * b_1(m_\pi = 0.4\,\mathrm{GeV})$
- The endpoint of the line of the first-order transition:

$b_1^c = 0.026$ ($m_\pi \approx 1.7\,\mathrm{GeV}$)

- Naive extrapolation of b_1 to the chiral limit $m_\pi \to 0$:

$b_1^{cl} \approx 0.2$

The expectation value of the Polyakov loop



- $b_1 = 0$ ($m_\pi = \infty$): first order phase transition
- $b_1 \geq 0.026$ ($m_\pi \geq 1.7$ GeV): cross over transition

Conclusion

- The $O(4)$-model lead to a stronger dependenc of T_c on m_π than seen in the lattice data. The reason is that the **chiral-symmetry is strongly broken**.

- We found the point where the line of first-order deconfinement phase transition ends at $m_\pi \approx 1.7$ GeV and $T \approx 240$ MeV.

- Going to smaller values of m_π (down to $m_\pi \approx 425$ MeV) the lattice data are well described by a **small explicit breaking of the $Z(3)$ symmetry**.