Hard parton damping in hot QCD

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– Hot Quarks 04, Taos, NM –

1. Motivation
2. Formalism
3. Implications

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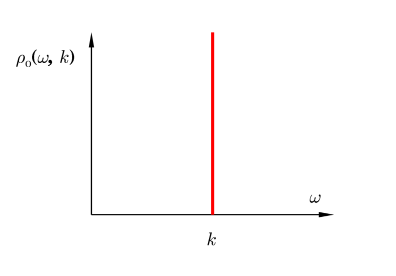
Propagator and spectral function

- Lehmann representation: $\Delta(k_0, k) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\rho(\omega, k)}{k_0 - \omega}$
  - $\rho(\omega) = \mathrm{Disc}\Delta(\omega)$
  - $\rho(\omega)$ odd, $\omega\rho(\omega) \geq 0$
  - sum rule $\int \frac{d\omega}{2\pi}\, \omega\rho(\omega, k) = 1$

- no interaction: $\rho(\omega, k) = \frac{1}{2k}[\delta(\omega - k) - \delta(\omega + k)]$



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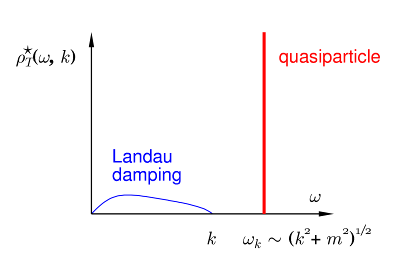
- weak coupling
  thermal masses $m \sim gT$
  (hard thermal loops)



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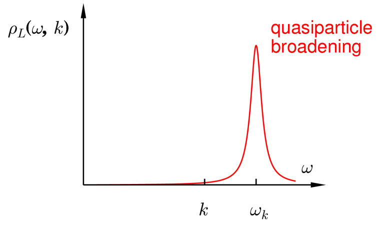
- collisions: non-zero width



Weak-coupling QCD

- thermal masses from on-shell 1-loop self-energies

$$
\begin{aligned}
m_g^2(T) &= \tfrac{1}{6}\left(N_c + \tfrac{1}{2} n_f\right) g^2 T^2 \\
m_q^2(T) &= \frac{N_c^2 - 1}{8 N_c}\, g^2 T^2
\end{aligned}
$$

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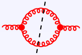
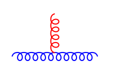
- width sensitive to screening $\Rightarrow$ (HTL) resummation required already at leading order [Braaten, Pisarski]
  - $k = 0$: $\gamma \sim g^2 T$
  - $k \neq 0$: $\gamma \sim g^2 T \ln g^{-1}$





sensitivity to soft magnetic sector of QCD (not yet fully understood)

  - $k \sim T$ (assuming $\gamma < m_{\rm mag} \sim g^2 T$, propagator has pole, ...)

$$\gamma = \frac{N_c}{8\pi} g^2 T \left(\ln \frac{\frac{2}{3}\, m_g^2}{m_{\rm mag}^2 + 2m_{\rm mag}\gamma} + 1.0968... \right)$$

Q: width (and masses) for $g^2(T) \gtrsim 1$ near $T_c \sim 200\,\text{MeV} \sim \Lambda_{\rm QCD}$

- extrapolating $\gamma \sim g^2 T \ln g^{-1}$: γ **important for larger** g
- Schwinger-Dyson: non-trivial (gauge inv., renormalization)
- lattice QCD: ???

Width near T_c

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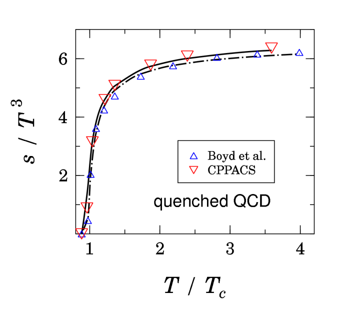
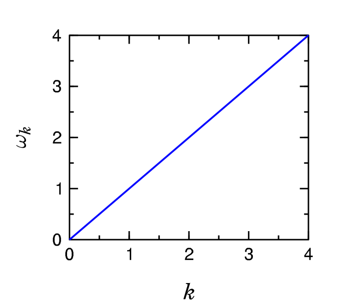
- **here:** constraints from known quantities sensitive to γ, m

entropy $\Leftrightarrow$ population of phase space





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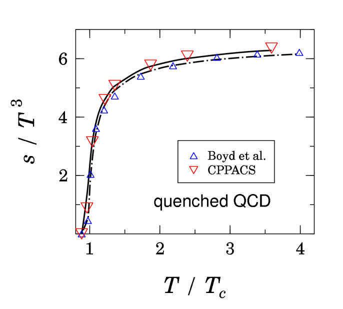

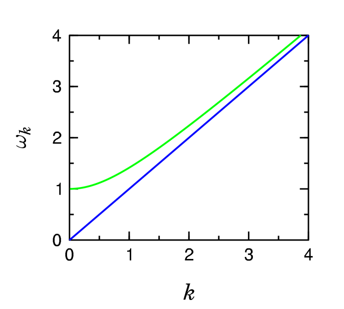

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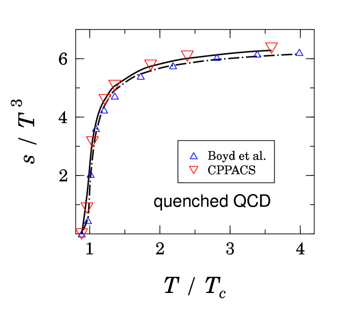
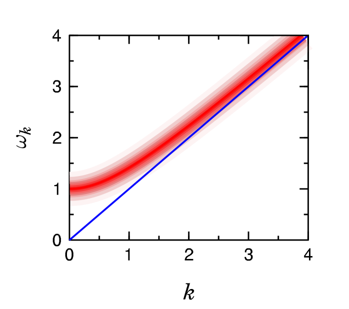
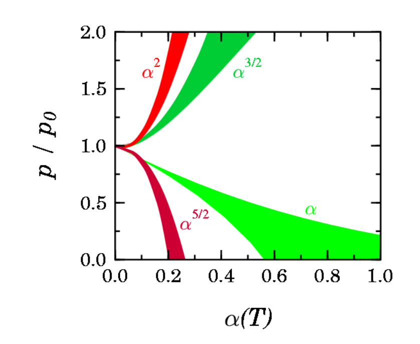




Thermodynamics at large coupling

- perturbation theory: not reliable near T_c
  'divergent' behavior at large coupling (cf. asymptotic series)

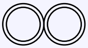
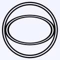


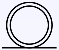
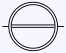
[Arnold et al.]

Thermodynamics at large coupling

- perturbation theory: not reliable near T_c
- resummations, Luttinger-Ward formalism (here φ^4 theory)

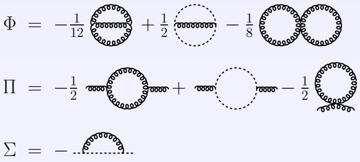
$$\Omega[\Delta] \;=\; \int_{k^4} n_B(\omega)\,\mathrm{Im}\Big(\ln(-\Delta^{-1}) + \Pi\Delta\Big) - \Phi[\Delta]$$

$\Phi[\Delta] \;=$  $+$  $+ \ldots$

$\Pi \;=\; 2\dfrac{\delta\Omega[\Delta]}{\delta\Delta} =$  $+$  $+ \ldots$

truncation of Φ: self-consistent approximations [Baym]
(non-trivial: renormalization)

Quasiparticle models in (quenched) QCD



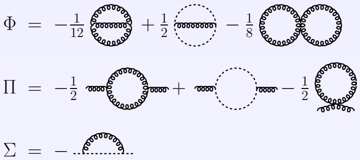
gauge invariance!

$\Downarrow$

relevant contributions

$\swarrow$

- transverse gluons ⇒ quasiparticles [Engels et al., Peshier et al., Heinz et al]
  - $m^2 = \Pi_t(\omega = k) \sim g^2 T^2$
  - effective coupling
- HTL quasiparticles [Blaizot et al., Peshier]
  - longitudinal modes
  - Landau-damping
  - pQCD running coupling

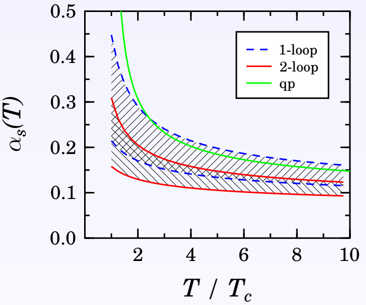


gauge invariance!

⇓

relevant contributions

⇙



- transverse gluons ⇒ quasiparticles [Engels et al., Peshier et al., Heinz et al]
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coupling large ... but ... width not taken into account so far

Dynamical quasiparticle entropy

- Luttinger-Ward formalism: $\delta\Omega/\delta\Delta = 0$

$$
\begin{aligned}
s &= -\frac{\partial\Omega}{\partial T} = -\int_{k^4} \frac{\partial n_B}{\partial T}\, \mathrm{Im}\Big(\ln(-\Delta^{-1}) + \Pi\Delta\Big) + \frac{\partial\Phi}{\partial T}\Big|_\Delta \\
&= -\int_{k^4} \frac{\partial n_B}{\partial T} \Big(\mathrm{Im}\ln(-\Delta^{-1}) + \mathrm{Im}\Pi\,\mathrm{Re}\Delta\Big) + \ldots
\end{aligned}
$$

- 2-loop resummed entropy: $s[\Delta] = s^{(0)} + \Delta s$

$$
s^{(0)} = T^{-1} \int_{k^3} \left[-T\ln\left(1 - e^{-\omega_k/T}\right) + \omega_k\, n(\omega_k)\right]
$$

'dispersion relation' ω_k from $\mathrm{Re}\,\Delta^{-1}(\omega_k) = 0$

$$
\Delta s = \int_{k^4} \frac{dn_B}{dT} \left[\mathrm{atn}\frac{\mathrm{Im}\Delta}{\mathrm{Re}\Delta} - \frac{\mathrm{Im}\Delta\,\mathrm{Re}\Delta}{(\mathrm{Re}\Delta)^2 + (\mathrm{Im}\Delta)^2}\right]
$$

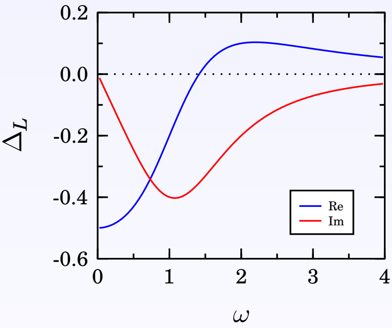
$\Uparrow$ contribution from non-trivial imaginary part of Δ

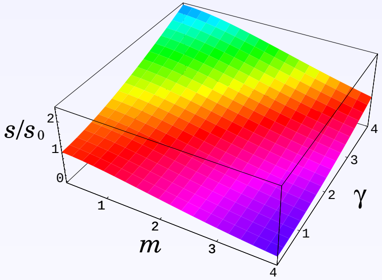
Lorentz spectral function

$$\rho_L(\omega) = \frac{\gamma}{E}\left(\frac{1}{(\omega-E)^2+\gamma^2} - \frac{1}{(\omega+E)^2+\gamma^2}\right)$$

$$\Delta_L(\omega,\mathbf{k}) = \frac{1}{\omega^2-\mathbf{k}^2-m^2+2i\gamma\omega} \qquad \omega_k = (k^2+m^2)^{1/2}$$

$$s_L(m,\gamma) = s^{(0)}(m) + \int_{k^4} \frac{\partial n}{\partial T}\left(\operatorname{atn}\frac{2\gamma\omega}{\omega_k^2-\omega^2} - 2\gamma\omega\frac{\omega_k^2-\omega^2}{(\omega^2-\omega_k^2)^2+(2\gamma\omega)^2}\right)$$





NB: $s_L(m=\gamma) = s_0$

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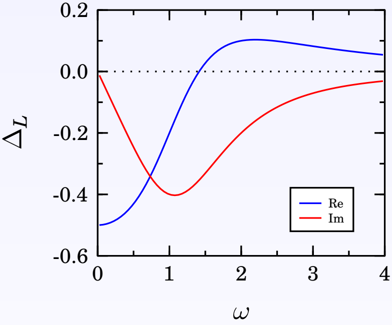
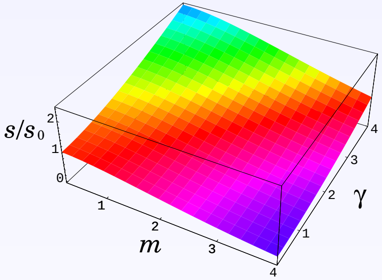




NB: $s_L(m=\gamma) = s_0$

Can in QCD γ be large near T_c?

General spectral functions

- assumption: $\rho(\omega)$ has peak at $E \approx \omega_k$ with char. width γ in Fourier space

$$\rho_f(t) = \frac{\sin Et}{iE}\, f(\gamma t)\,, \quad f(0) = 1 \ \ \text{(sum rule)}$$

retarded propagator by 'forward' Fourier transform

$$\Delta_f(k_0) = \frac{i}{2E} \int_0^\infty dt\, e^{ik_0 t} \left(e^{iEt} - e^{-iEt} \right) f(t)$$

- resulting entropy:
  - increase with γ
  - sensitive to long-time behavior of $f(\gamma t)$ ($s_L(m, \gamma)$, from $f_L = \exp(-\gamma |t|)$, is representative)
  - relevant: dispersion relation and width at large momenta

QCD quasiparticles with width

- entropy for Lorentz spectral function (quenched limit)

$$s^{qQCD} = 2(N_c^2 - 1)\, s_L(m, \gamma)$$

- ansatz for mass and width

$$m^2 = \frac{N_c}{6}\, g^2 T^2$$

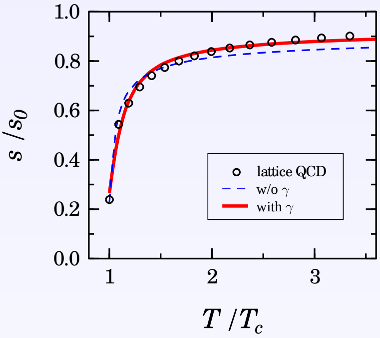
$$\gamma = \frac{3}{4\pi}\, \frac{m^2}{T^2}\, T \ln \frac{c}{(m/T)^2}$$

NB: relation between γ, m fixed; c not just a 3rd fit parameter

- effective coupling with 2 fit parameters

$$g^2(T) = \frac{48\pi^2}{11 N_c \ln(\lambda(T - T_s)/T_c)^2}$$

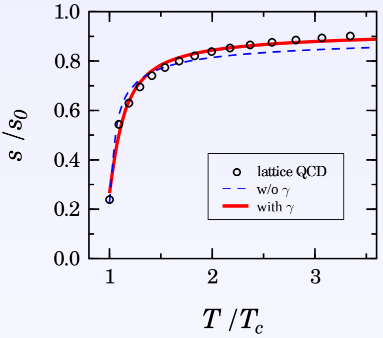
QCD ‘quasiparticles’

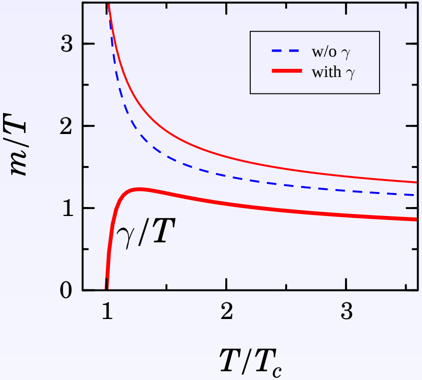


better fits than w/o width

lattice data (quenched): [CPPACS]

QCD 'quasiparticles'





better fits than w/o width

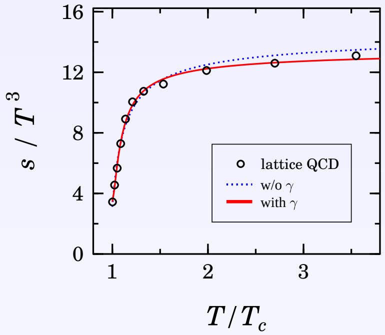
lattice data (quenched): [CPPACS]

- $T > 1.2\,T_c$: $\gamma \sim m$
- $T < 1.2\,T_c$: $\gamma \ll m$

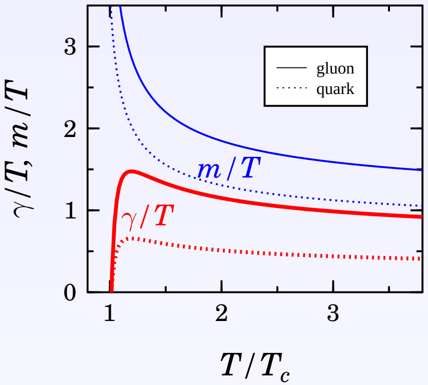
here from factor $\ln c/(m/T)^2$

also from *ansatz* $\gamma \sim g^2 T e^{-ag}$

QCD with $n_f = 2$ light flavors



lattice data: [Karsch et al.]



universal for QCD near T_c:

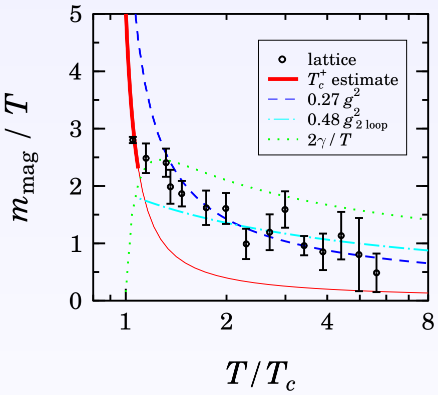
entropy small $\Rightarrow$ m large, γ small

Magnetic mass

- near T_c: relate c to Pisarski's nlo. result (derived for $\gamma \ll m_{\rm mag}$)

$$m_{\rm mag}(T_c) \approx \sqrt{\frac{2}{c}}\,\frac{m(T_c)^2}{T_c} \approx 5\,T_c$$

- larger T: *ansatz* $m_{\rm mag} \propto g^2 T$ with fitted coupling

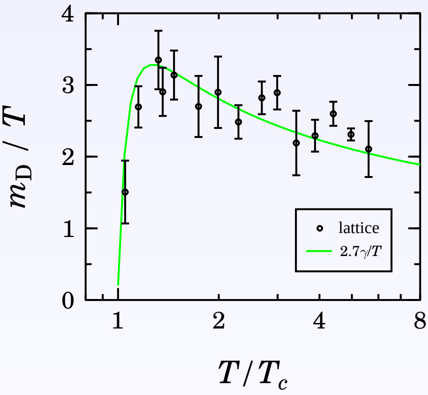


lattice data (quenched):
[Nakamura et al.]

Debye mass

- empirical observation for Debye mass near T_c

$$m_{\rm D}(T) \propto \gamma(T)$$

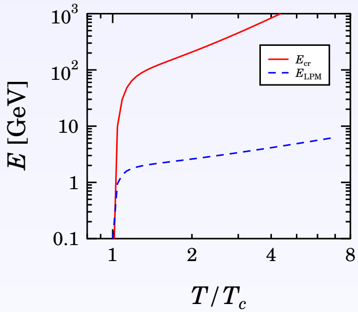


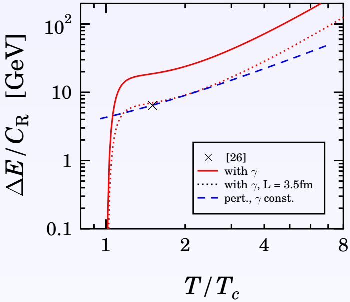
lattice data (quenched):
[Nakamura et al.]

energy loss of hard jet in quark-gluon plasma of length L, by radiating gluons with energy ω

- LPM regime $\quad m_{\mathrm{D}}^2/\gamma = E_{LPM} < \omega < E_{cr} = m_{\mathrm{D}}^2 \gamma L^2$

$$\Delta E = -\frac{C_R}{8}\,\alpha\, m_{\mathrm{D}}^2 \gamma L^2 \ln(\gamma L) \qquad \text{[BDMPS]}$$

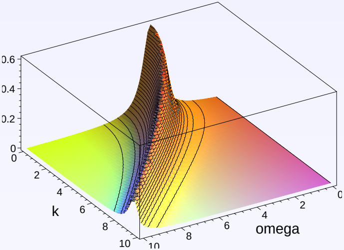


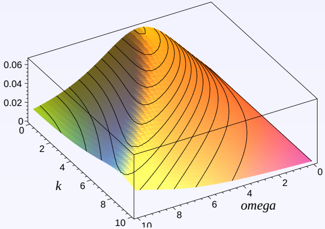


- at $T \approx 1.2\,T_c$: significant change

Resumé

- width has significant effect on thermodynamic bulk properties (except for exotic spectral functions)
- QCD for $T > T^\star$: broad excitations
  $T_c < T < T^\star$: heavy narrow modes (quasiparticles)





- characteristic temperature $T^\star \approx 1.2\,T_c$ – **observable!?**

details: A. Peshier, hep-ph/0403225 (to be published in PRD)